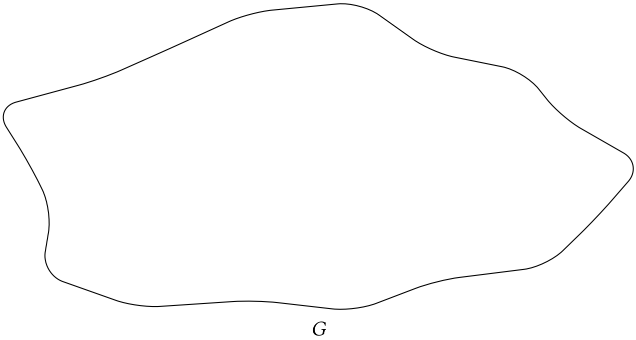
On the Minimum Eccentricity Shortest Path Problem

Feodor Dragan and Arne Leitert

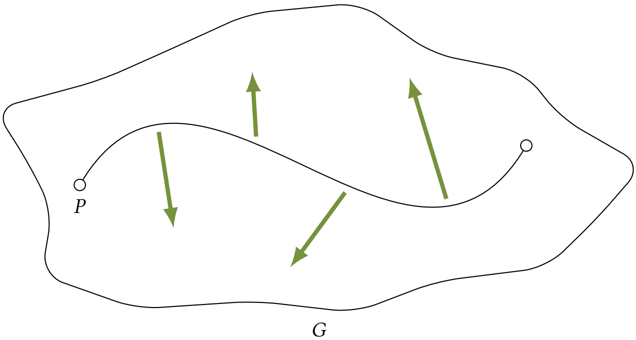
Minimum Eccentricity Shortest Path

Given a graph G.



Minimum Eccentricity Shortest Path

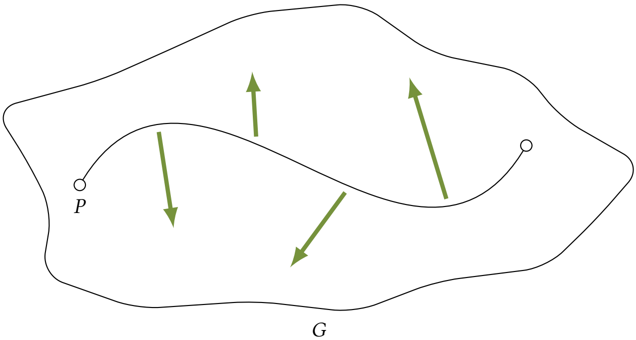
Find a shortest path P with minimum eccentricity,
i. e., minimise $\max_{v \in V} d(v, P)$



Minimum Eccentricity Shortest Path

Find a shortest path P with minimum eccentricity,
i. e., minimise $\max_{v \in V} d(v, P)$

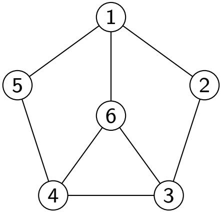
Also called *k-Dominating Shortest Path*



Motivation

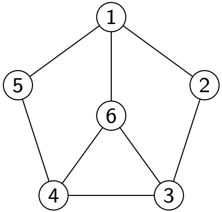
Line-Distortion

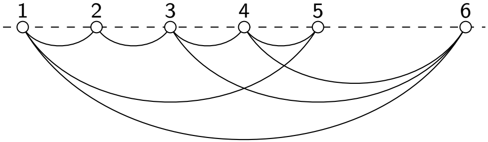
Given a graph $G = (V, E)$



Line-Distortion

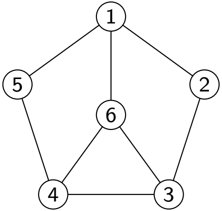
Find an injective function $f\colon V \to \mathbb{N}$ with $d(u,v) \leq |f(u) - f(v)|$.

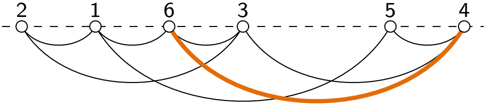




Line-Distortion

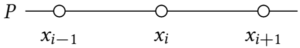
Line-distortion $\mathrm{ld}(G) = \min_{f} \max_{uv \in E} |f(u) - f(v)|$.





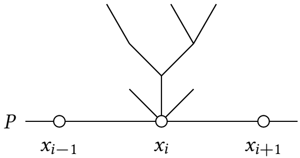
Approximating Line-Distortion

Assume G has a shortest path P with eccentricity k.



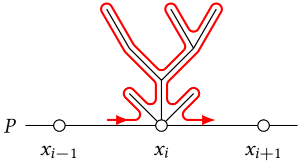
Approximating Line-Distortion

Build BFS-tree T from P.



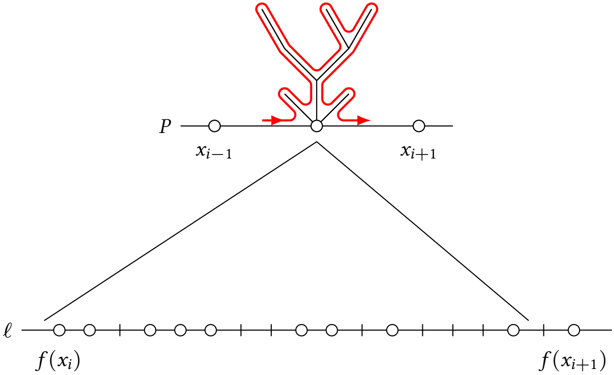
Approximating Line-Distortion

Perform preorder traversal on T.



Approximating Line-Distortion

Embed vertices into line ℓ as visited during traversal.



Approximating Line-Distortion

G has shortest path P of eccentricity k and $\mathrm{ld}(G) = \lambda$

- Embedding is $(8k + 2)$-approximation
- In linear time if P is given.

- $k \leq \lfloor \lambda/2 \rfloor$
- In some cases: $\lambda - k \approx n$

Conclusion

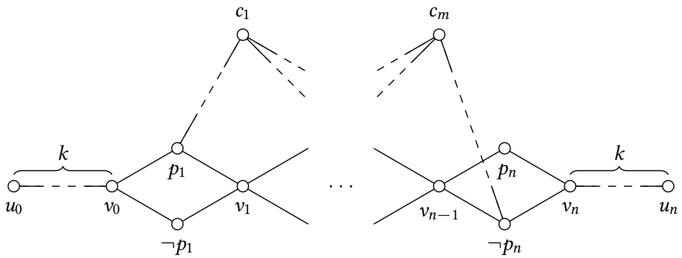
- Reproducing existing results if $\lambda \approx k$.
- Stronger result if $\lambda - k \approx n$.
- Fast approximation for MESP leads to fast approximation for LD

General Results

NP-Completeness

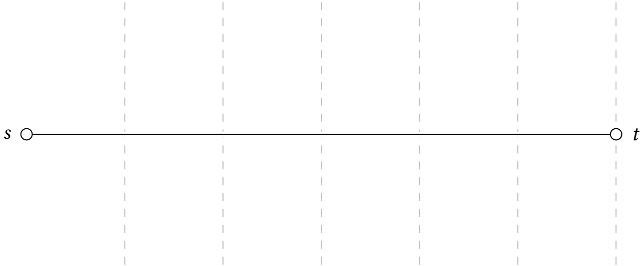
NP-Complete

- Reduction from SAT
- also NP-c. if
  - s and t are given
  - vertex degree is limited to 3 (by V. B. Le, University of Rostock)



2-Approximation

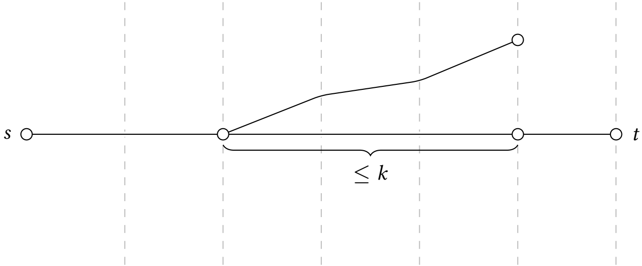
Consider a shortest (s, t)-path with eccentricity k and a BFS(s)-layering



2-Approximation

Observation:

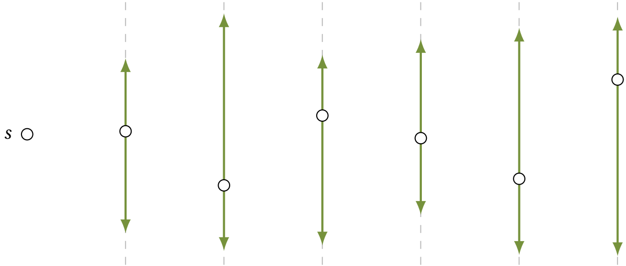
- Each layer has radius at most $2k$.



2-Approximation

Algorithm:

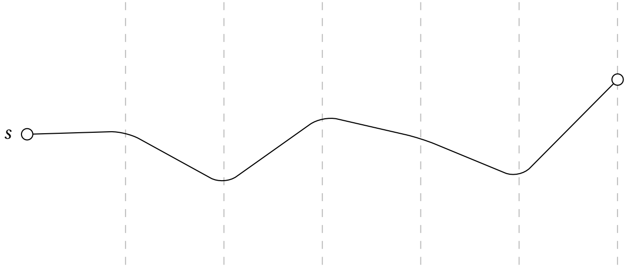
- Determine *layer-wise eccentricity* for each vertex v.
- Pick path where max. layer-wise eccentricity is minimal. (modified BFS)



2-Approximation

Runtime:

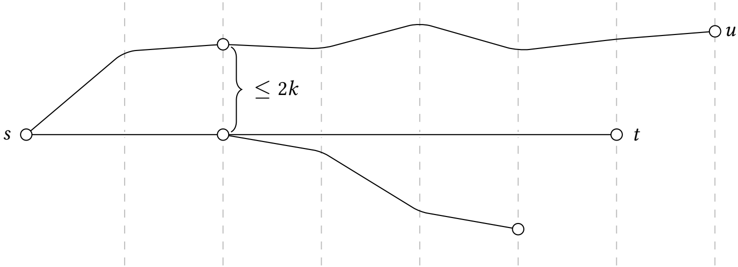
- $\mathcal{O}(n^3)$ for all s
- $\mathcal{O}(nm)$ if s is given



3-Approximation

Observation:

- Each shortest (s, u)-path with $d(s, t) \leq d(s, u)$ has eccentricity $\leq 3k$.



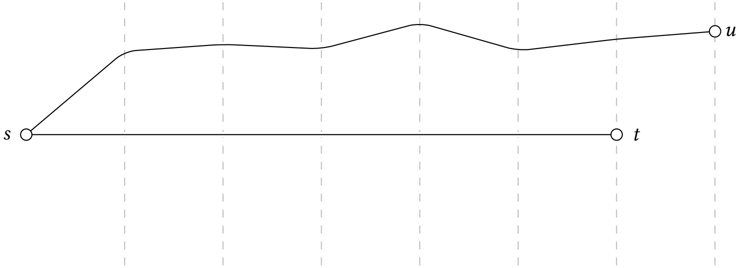
3-Approximation

Algorithm:

- Find a shortest path to a vertex u for which $d(s, u)$ is maximal.

Runtime

- $\mathcal{O}(nm)$ for all s
- $\mathcal{O}(m)$ for a given s



Other Results

Approximation

- 8-approximation in linear time

Exact solution

- Check if $k = 1$ in $\mathcal{O}(n^3 m)$ time.
- Determine k in $\mathcal{O}(n^{2k+2} m)$ time.

k-Domination

- If k is known, a k-dominating set can be found in $n^{\mathcal{O}(k)}$ time.

Special Classes

Distant-Hereditary Graphs

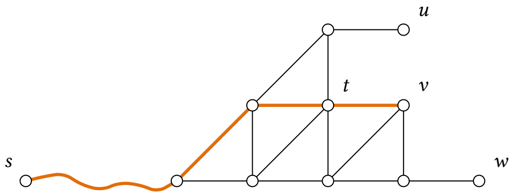
If x, y is a diametral pair, then there is a shortest (x, y)-path with eccentricity k.

- Very simple linear time algorithm for trees (two BFS calls)
- Linear time algorithm for distant-hereditary graphs

Chordal Graphs

Not necessarily diameter

- Diameter; $s, \dots, w$
- Optimal path: $s, \dots, t, v$



For given s, t pair: $\mathcal{O}(nm)$ time algorithm.

Open Questions

How hard is finding s and t?

- Our approaches often iterate over all s, t-pairs (or at least all s).
- Problem remains NP-complete if s ant t is given.

Other graph classes

- Planar?
- Graphs without tree structure?