Line-Distortion, Bandwidth and Path-Length of a Graph

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Authors

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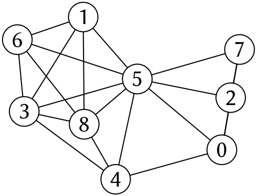
Presenter

Line-Distortion and Bandwidth

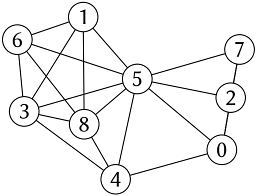
Bandwidth and Line-Distortion

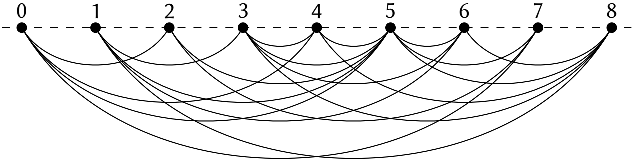
Given a graph $G = (V, E)$



Bandwidth and Line-Distortion

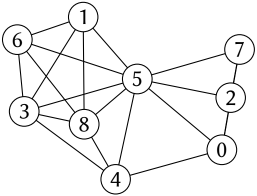
Find an injective function $f\colon V \to \mathbb{N}$.

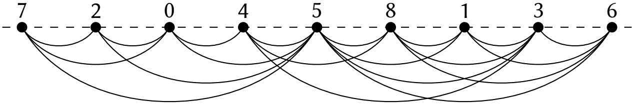




Bandwidth and Line-Distortion

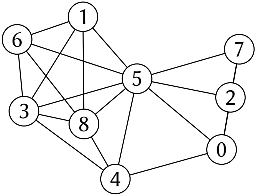
Optimize f such that $\max_{uv \in E} |f(u) - f(v)|$ is minimal.

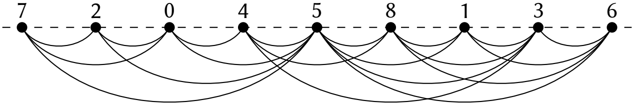




Bandwidth and Line-Distortion

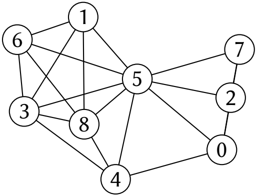
$$\mathrm{bw}(G) = \min_{f} \max_{uv \in E} |f(u) - f(v)|$$

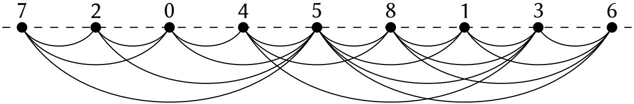




Bandwidth and Line-Distortion

Additional for line-distortion $\mathrm{ld}(G)$: $d_G(u, v) \leq |f(u) - f(v)|$ for all $u, v \in V$.





Bandwidth and Line-Distortion

Both problems are very hard.

Graph Class	Solution Quality	Time	Source
Trees	$\mathcal{O}(1)$-approx.	NP-hard	Blache et al. 1997
Caterpillars	$\mathcal{O}(1)$-approx.	NP-hard	Dubeya et al. 2011
(hair-length ≤ 2)	optimal	$\mathcal{O}(n \log n)$	Assman et al. 1981
(hair-length ≤ 3)	optimal	NP-hard	Monien 1986
Convex Bipartite	optimal	NP-hard	Shrestha et al. 2012
Interval	optimal	$\mathcal{O}(n \log^2 n)$	Sprague 1994
Chordal	$\mathcal{O}(\log^{2.5} n)$-approx.	polynomial	Gupta 2001

Some bandwidth results.

Bandwidth and Line-Distortion

Both problems are very hard.

Graph Class	Solution Quality	Time	Source
General	optimal	$\mathcal{O}(n\lambda^4(2\lambda+1)^{2\lambda})$	Fellows et al. 2009
	$\mathcal{O}(n^{1/2})$-approx.	polynomial	Bădoiu et al. 2005
Trees	$\mathcal{O}(n^{1/3})$-approx.	polynomial	Bădoiu et al. 2005
Bipartite	optimal	NP-hard	Heggernes et al. 2010
Cocomparability	optimal	NP-hard	Heggernes et al. 2010
	6-approx.	$\mathcal{O}(n \log^2 n + m)$	Heggernes et al. 2010
split	optimal	NP-hard	Heggernes et al. 2010
	6-approx.	linear	Heggernes et al. 2010

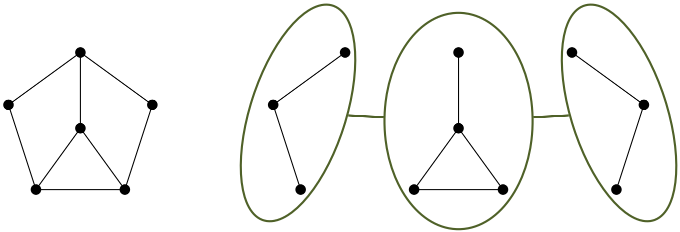
Some line-destortion results.

Our Approach: Path-Length

Path Decomposition and Path-Length

Sequence of subsets of V called bags

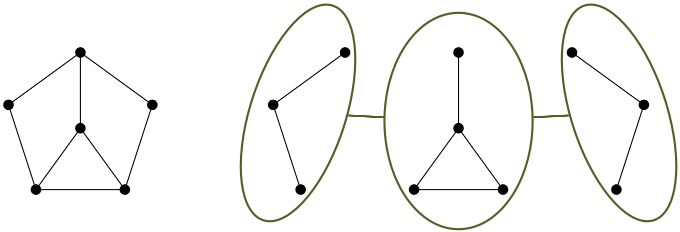
- Each vertex is in a bag.
- Each edge is in a bag.
- Each vertex induces a subpath.



Path Decomposition and Path-Length

Path-Length $\mathrm{pl}(G) = \lambda$:

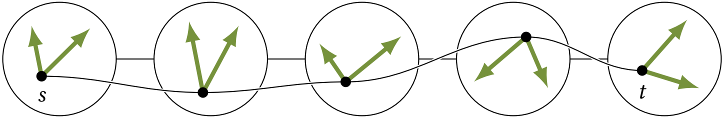
- Smallest maximal diameter of all decompositions is at most λ



Dominating Path

If $\mathrm{pl}(G) = \lambda$, G has λ-dominating shortest path.

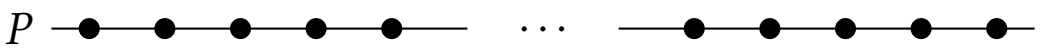
- Vertex s in first bag.
- Vertex t in last bag.
- Path s to t is λ-dominating.



Algorithm

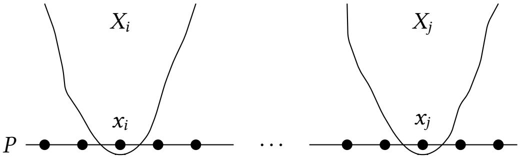
Algorithm

Find a λ-dominating shortest path $P = (x_0, x_1, \ldots, x_q)$.



Algorithm

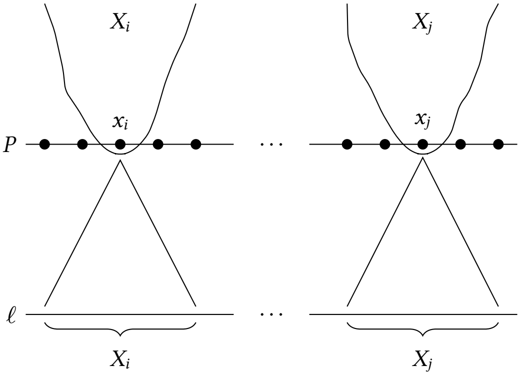
Partition V into sets $X_0, X_1, \ldots, X_q$ based on a $BFS(P)$-tree.



Algorithm

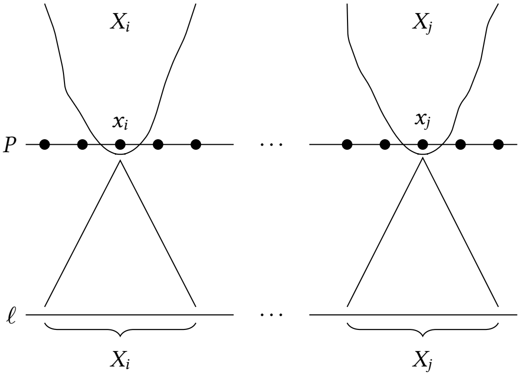
Create an embedding f into a line ℓ.

- Placing vertices of X_i before all vertices of X_j, $i < j$.
- Embed X_i as described by Bădoiu et al. 2005.
  (Simple linear time algorithm)



Algorithm

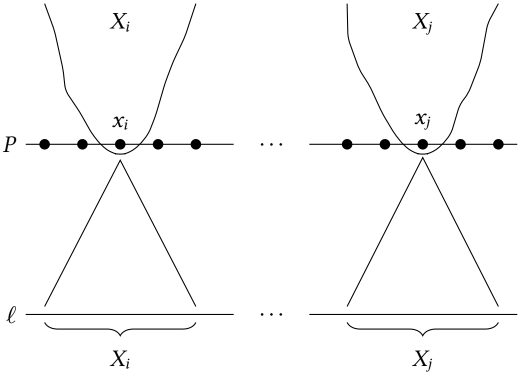
For line-distortion: Leave a space of length $2k+1$ between X_i and X_{i+1}.



Algorithm

Approximation factor for a graph G with $\mathrm{pl}(G) = \lambda$

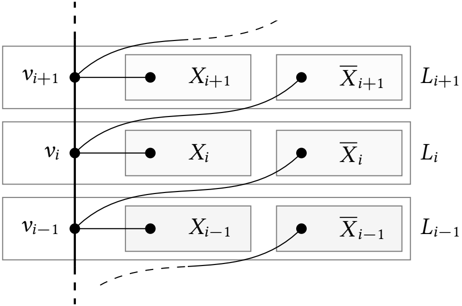
- Bandwidth: $4\lambda + 2$
- Line-Distortion: $12\lambda + 7$



Other Results

AT-free Graphs

- path-length at most 2
- 8-approx. for line-distortion in linear time.



Other Results

AT-free Graphs

- path-length at most 2
- 8-approx. for line-distortion in linear time.

Graphs with path-length λ

- Finding a decomposition with length 2λ in $\mathcal{O}(n^3)$ time.
- Finding a λ-dominating shortest path in $\mathcal{O}(nm)$ time.
- Finding a 2λ-dominating shortest path in $\mathcal{O}(n+m)$ time.

Thank You!