Efficient Dominating and Edge Dominating Sets for Graphs and Hypergraphs

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The Problem

Domination

$$D \subseteq V \text{ with } \bigcup_{d \in D} N[d] = V$$

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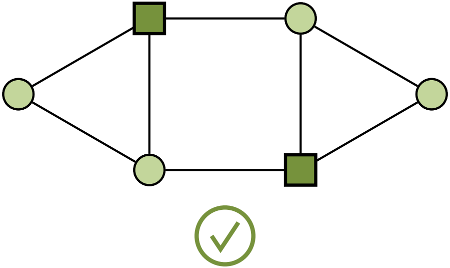

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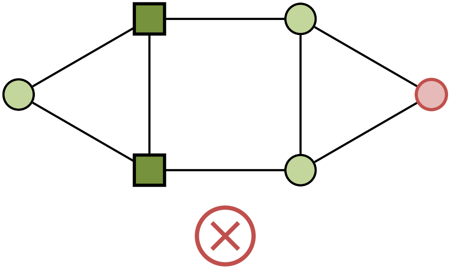

Efficient Domination

Exactly one $d \in D$ for each $v \in V$

$\forall\, v \in V : \exists!\, d \in D : v \in N[d]$

Exact cover of the closed neighbourhoods

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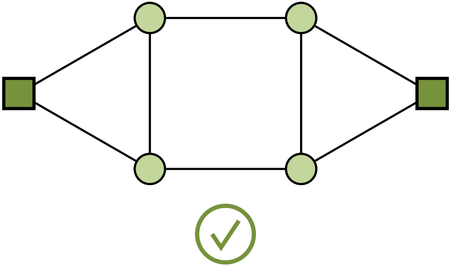

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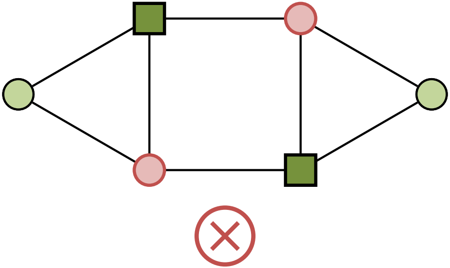

Efficient Domination

packing and covering problem

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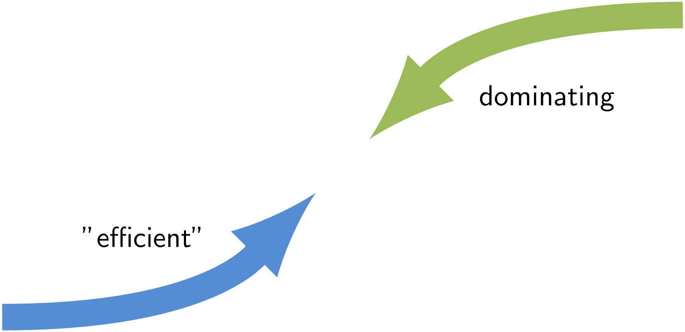

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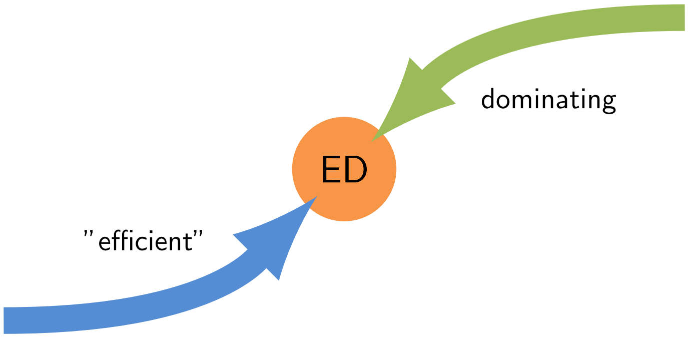
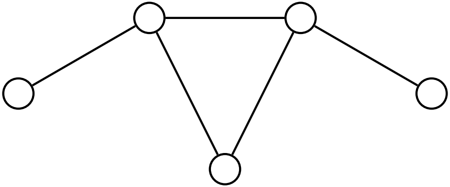


Efficient Domination

Does not always exist



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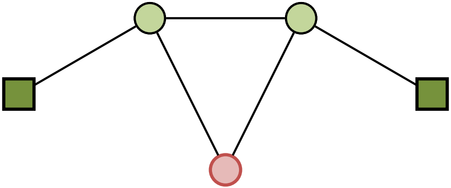

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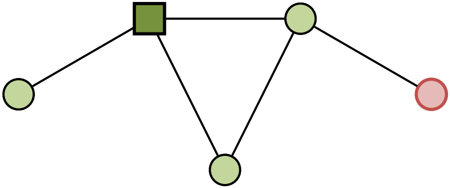

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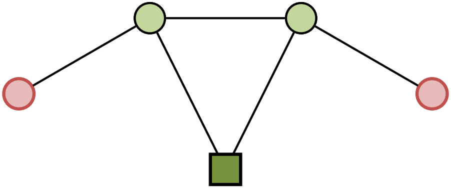
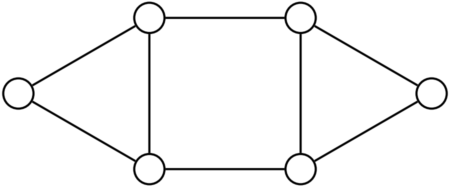


Efficient Edge Domination

the same with edges



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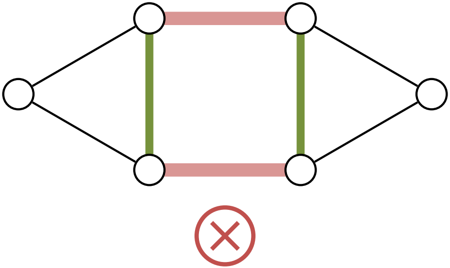

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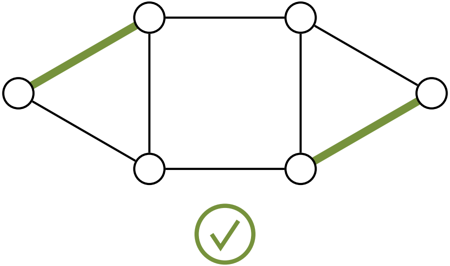


Other Names

Efficient Domination

- Independent Perfect Domination

Efficient Edge Domination

- Dominating Induced Matching

Existing Results

ED

co-comparability	linear	Chang et al 1995
planar bipartite, chordal bipartite	NP-c.	Lu, Tang 2002
chordal	NP-c.	Yenn, Lee 1996

EED

P_7-free	linear	ISAAC 2011
planar bipartite	NP-c.	Lu, Ko, Tang 2002
chordal	linear	Lu, Ko, Tang 2002

and more...

Our Solution

What is known

By Definition

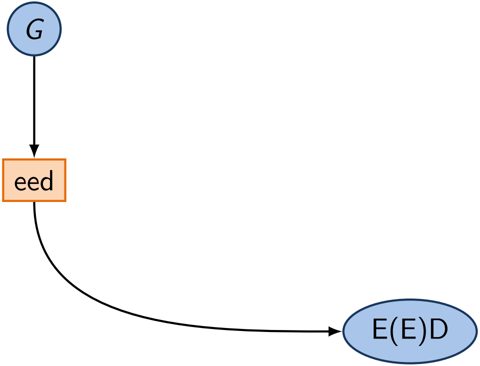
The following are equivalent for any D:

(i) D is EED in G

(ii) D is ED in $L(G)$

(iii) D is dominating set in $L(G)$ and independent set in $L(G)^2$

What is known



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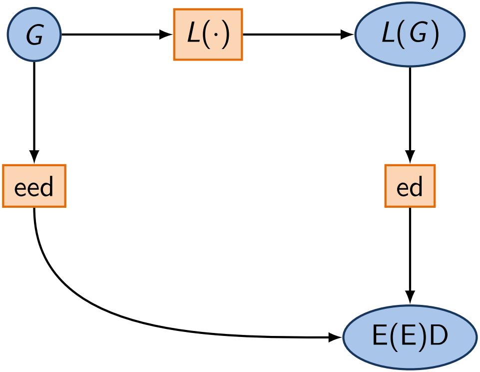

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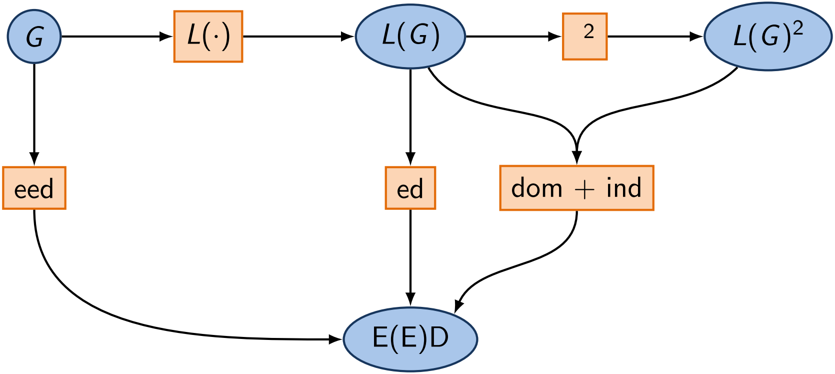

Our Approach

Vertex weight function $\omega : V \to \mathbb{N}$ with $\omega(v) = |N[v]|$

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Theorem

The following are equivalent for any $D \subseteq V$:

(i) D is ED in G

(ii) D is min. w. dominating set in G with $\omega(D) = |V|$

(iii) D is max. w. independent set in G^2 with $\omega(D) = |V|$

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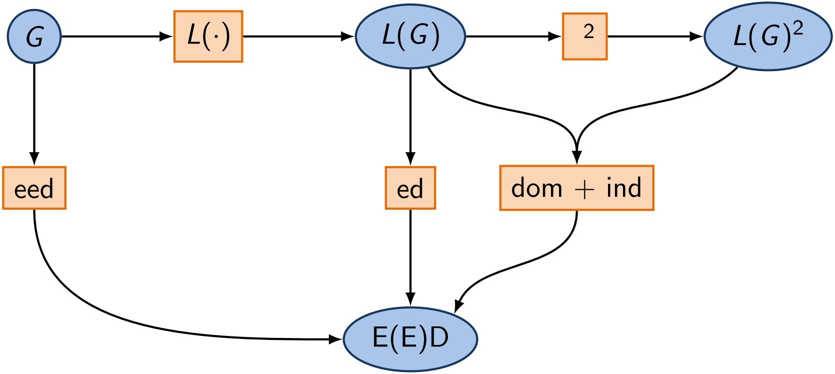
(i) $\Leftrightarrow$ (iii) was also found by Martin Milanič.

Our Approach



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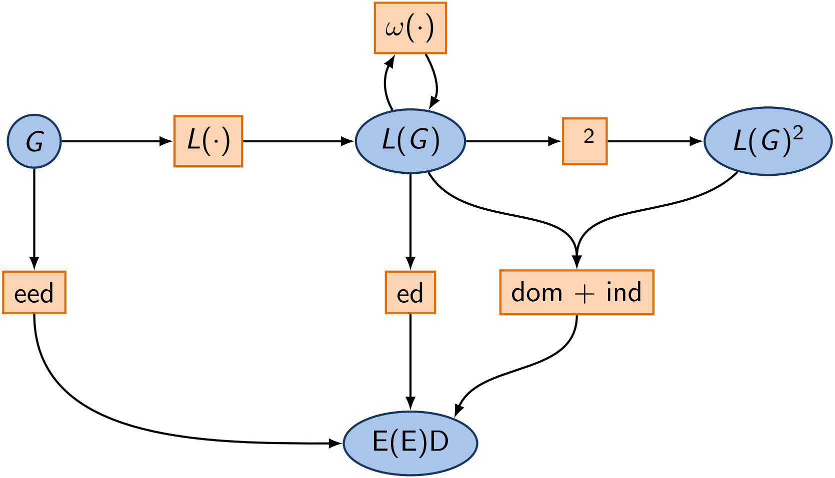

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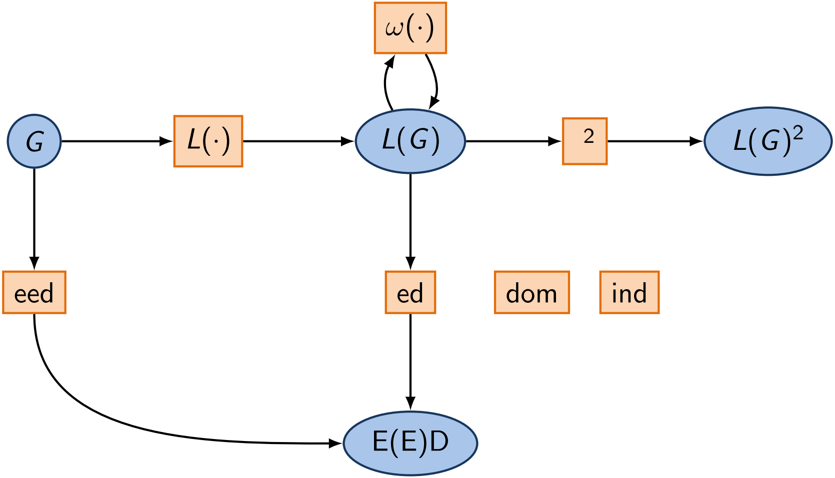

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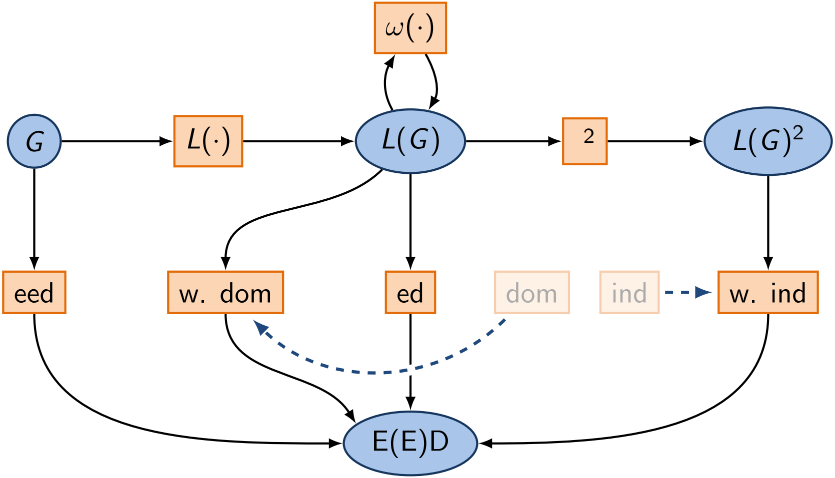

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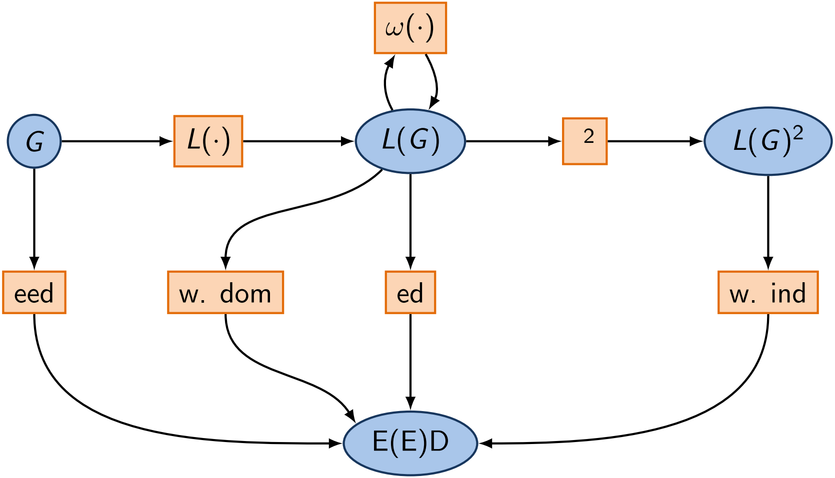

This includes ...

... for EED

- min. w. (perfect, independent) edge domination
- min. w. dominating matching
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... for ED

- min. w. perfect domination
- min. w. independent domination

Results

dually chordal

- ED: linear
  using independent set for G^2
- EED: linear
  G has EED $\Rightarrow$ dually chordal $\leftrightarrow$ chordal

This includes strongly chordal graphs.

AT-free

- ED: polynomial
  using independent set for G^2
  (G^2 is AT-free)

- EED: polynomial
  using independent set for $L(G)^2$
  ($L(G)^2$ is AT-free)

Results

interval bigraphs

- ED: polynomial
  using domination for G

Results

interval-filament

- EED: polynomial
  using independent set for $L(G)^2$
  ($L(G)^2$ is interval-filament)

Results

weakly chordal

- EED: polynomial
  using independent set for $L(G)^2$
  ($L(G)^2$ is weakly chordal)

Results

	ED	EED
dually chordal	linear	linear
AT-free	polynomial	polynomial
interval bigraphs	polynomial	
interval-filament		polynomial
weakly chordal		polynomial

A View on Hypergraphs

Hypergraphs

Hypergraph: $H = (V, \mathcal{E})$ with $\mathcal{E} \subseteq \wp(V) \setminus \emptyset$

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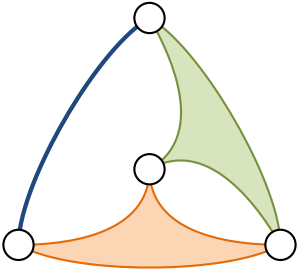

Hypergraphs

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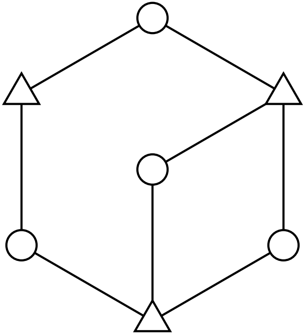

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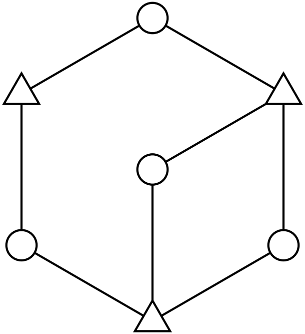
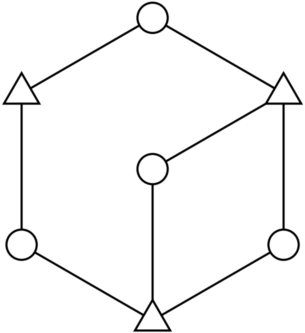


incidence graph of H — $\mathcal{I}(H)$

Duality

Hypergraph

$H = (\bigcirc, \triangle)$



$\mathcal{I}(H)$

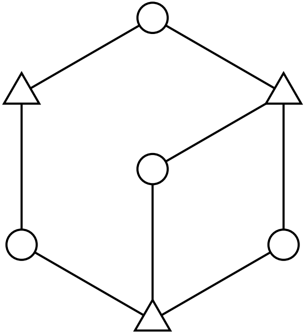
Duality

Hypergraph

$H = (\bigcirc, \triangle)$

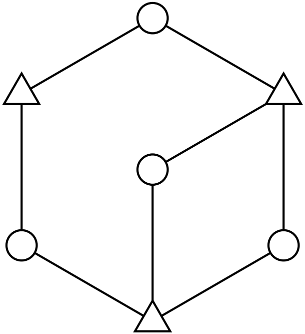
Dual Hypergraph

$H^* = (\triangle, \bigcirc)$



$\mathcal{I}(H) = \mathcal{I}(H^*)$

Line / 2-Section graph



$\mathcal{I}(H)$

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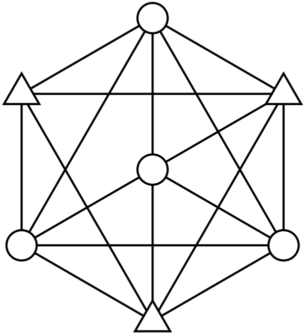
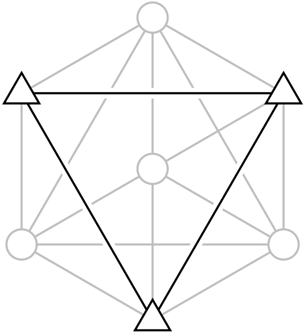


$\mathcal{I}(H)^2$

line graph

$L(H) = \mathcal{I}(H)^2[\mathcal{E}]$



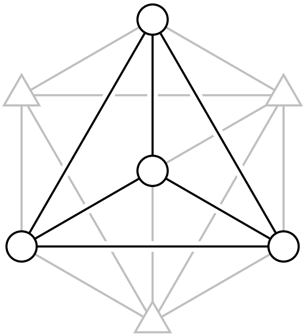
Line / 2-Section graph

line graph

$L(H) = \mathcal{I}(H)^2[\mathcal{E}]$

2-Section graph

$2Sec(H) = \mathcal{I}(H)^2[V]$



$2Sec(H)$

Efficient (Edge) Domination

Definition on Hypergraphs

D is ED in $H \Leftrightarrow D$ is ED in $2Sec(H)$.
D is EED in $H \Leftrightarrow D$ is ED in $L(H)$.

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Theorem

H has an ED $\Leftrightarrow H^*$ has an EED.

Results on hypergraphs

	ED	EED
α-acyclic	NP-complete	polynomial
hypertree	polynomial	NP-complete

Thank you for your attention!